

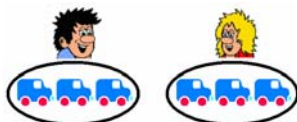
Division

For children to understand division they need to experience two types of activity: **grouping and sharing**. Pupils should experience the different types of division in a wide range of practical, relevant contexts. Children are generally introduced to division through practical activities that initially involve sharing, and later grouping, of objects.

Sharing

- **Equal sharing** occurs when a quantity is shared out equally into a given number of portions, and we work out how many there are in each portion.
- When we share we know how many we have to share out and how many to share between but not how many they will each get.

6 toy cars are shared between 2 children. How many will they have each?



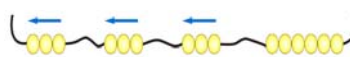
Grouping

- **Grouping** occurs when we are asked to find how many groups of the divisor are in the original amount.
- We know how many we have and how many to put into each 'set' but not the number of 'sets' we will have.

There are 6 cars; each child can have 2 cars. How many children will get 2 cars?



15 beads put into groups of 3.



Mental methods for division - Partitioning

Introduced in Year 3 and developed into Year 4

Partitioning may be used initially as a mental method for division, progressing to being used alongside the development of short division.

It involves partitioning the dividend into a multiple of the divisor, plus the remaining ones enabling each part to be divided with greater ease.

E.g. $84 \div 7$ may be calculated by partitioning the 84 into a **multiple of the divisor** and the **remaining number** to be **divided separately**. Results are then added to find the answer (**quotient**).

Informal recording

$$\begin{array}{r} 84 \\ 70 + 14 \\ \downarrow \quad \downarrow \quad \div 7 \\ 10 + 2 = 12 \end{array}$$

In this example, using knowledge of multiples, the 87 is partitioned into 60 (the highest multiple of 3 that is also a multiple of 10 and less than 87) plus 27.

Each part is divided separately using the distributive law.

Results are then added to find the answer.

Recording mental division using partitioning

$$\begin{aligned} 87 \div 3 &= (60 + 27) \div 3 \\ &= (60 \div 3) + (27 \div 3) \\ &= 20 + 9 \\ &= 29 \end{aligned}$$

Children should be able to find a **remainder** mentally, for example the remainder when 34 is divided by 6.

Remainders after division can be recorded similarly

$$\begin{aligned} 96 \div 7 &= (70 + 26) \div 7 \\ &= (70 \div 7) + (26 \div 7) \\ &= 10 + 3 \text{ r } 5 \\ &= 13 \text{ r } 5 \end{aligned}$$

Written methods of division

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division



Year 5 / 6 division

Short division of $TU \div U$	Linked to partitioning
<ul style="list-style-type: none"> 'Short' division of $TU \div U$ can be introduced as a more compact recording of the mental method of partitioning. Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7. 	<p>For $81 \div 3$, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give $60 + 21$.</p> <p>Each number is then divided by 3.</p> $81 \div 3 = (60 + 21) \div 3$ $= (60 \div 3) + (21 \div 3)$ $= 20 + 7$ $= 27$ <p>The short division method is recorded like this:</p> $\begin{array}{r} 20 + 7 \\ 3 \overline{) 60 + 21} \end{array}$ <p>The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.</p> <p>The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.</p>
<ul style="list-style-type: none"> 'Short' division of $HTU \div U$ can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. The accompanying patter is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. 	<p>For $291 \div 3$, because $3 \times 90 = 270$ and $3 \times 100 = 300$, we use 270 and split the dividend of 291 into $270 + 21$. Each part is then divided by 3.</p> $291 \div 3 = (270 + 21) \div 3$ $= (270 \div 3) + (21 \div 3)$ $= 90 + 7$ $= 97$ <p>The short division method is recorded like this:</p> $\begin{array}{r} 90 + 7 \\ 3 \overline{) 270 + 21} \end{array}$

Written methods of division - 'chunking'

Year 5 into Year 6

'Expanded' method for $HTU \div U$ Linked to repeated subtraction

- This method is based on subtracting multiples of the **divisor** from the number to be divided, the **dividend**.
- For $TU \div U$ there is a link to the mental method.
- As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'
- Once they understand and can apply the method, children should be able to move on from $TU \div U$ to $HTU \div U$ quite quickly as the principles are the same.
- This method, often referred to as '**chunking**', is based on **subtracting multiples of the divisor, or 'chunks'**. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding children of the link between division and repeated subtraction.
- However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

$$97 \div 9$$

$$\begin{array}{r} 9 \overline{)97} \\ - 90 \quad \times 10 \\ \hline 7 \end{array}$$

Answer: 10 r 7

$$\begin{array}{r} 6 \overline{)196} \\ - 60 \quad \times 10 \\ \hline 136 \\ - 60 \quad \times 10 \\ \hline 76 \\ - 60 \quad \times 10 \\ \hline 16 \\ - 12 \quad \times 2 \\ \hline 4 \end{array}$$

Answer: 32 r 4

- The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for $HTU \div U$ involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.
- Estimating has two purposes when doing a division:
 - to help to choose a starting point for the division;
 - to check the answer after the calculation.
- Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

To find $196 \div 6$, we start by multiplying 6 by 10, 20, 30, ... to find that $6 \times 30 = 180$ and $6 \times 40 = 240$. The multiples of 180 and 240 trap the number 196. This tells us that the answer to $196 \div 6$ is between 30 and 40.

Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.

$$\begin{array}{r} 6 \overline{)196} \\ - 180 \quad \times 30 \\ \hline 16 \\ - 12 \quad \times 2 \\ \hline 4 \end{array}$$

Answer: 32 r 4

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Long division

- The next step is to tackle $HTU \div TU$, which for most children will be in Year 6.
- The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.
- Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

Linked to repeated subtraction

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As $24 \times 20 = 480$ and $24 \times 30 = 720$, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{r} 24 \overline{)560} \\ \underline{-480} \quad \times 20 \\ 80 \\ \underline{-72} \quad \times 3 \\ 8 \end{array}$$

Answer: 23 r 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} 23 \\ 24 \overline{)560} \\ \underline{-480} \quad \times 20 \\ 80 \\ \underline{-72} \quad \times 3 \\ 8 \end{array}$$

Answer: 23 r 8