

Subtraction

- Story around structure - I have a set of 6 objects to start with and so many of them get partitioned off, taken away, eaten, lost... 'How many are left?'
- Often modelled with sets of 'things' - essentially the story follows the same plot of 'have', 'take away', 'have left'
- Lots of songs and rhymes.
- Very practical.

Early Skills

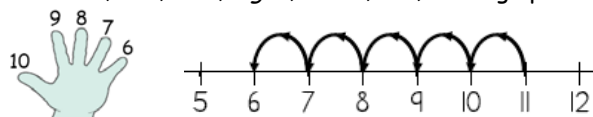
Count out - a child finding $9 - 3$, gets 9 objects and takes away 3, counts how many remain.



Count back from - a child finding $9 - 3$, counts back three numbers from 9: 'eight, seven, six'.



Count back to - a child doing $11 - 6$, counts back from the first number to the second, keeping a tally using fingers of the number of numbers that have been said, 'ten, nine, eight, seven, six', holding up five fingers.



Count up - a child doing $11 - 6$, counts up from 6 to 11, 'seven, eight, nine, ten, eleven', sometimes keeping a count of the spoken numbers using fingers.



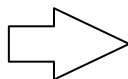
Partitioning - It is really important to stress the fact that we only partition the second number.

Using a hundred square

- count back the tens and then the ones

Using Tens and Units apparatus

- take away the tens and then the ones
(ensure children are confident in exchanging a 10 rod for 10 units to enable them to cross the tens boundary)



The practical resources should be used alongside the written partitioning method.

Where no boundary is crossed

$$\begin{aligned} \text{E.g. } 58 - 23 &= 58 - 20 - 3 \\ &= 38 - 3 \\ &= 35 \end{aligned}$$

Where a boundary is crossed

$$\begin{aligned} \text{E.g. } 53 - 28 &= 53 - 20 - 8 \\ &= 33 - 8 \\ &= 25 \end{aligned}$$

Early Number line - moving from marked to empty number lines (see supportive documentation)

Counting back

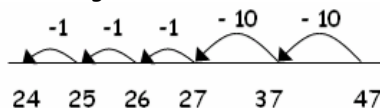
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

$$15 - 7 = 8$$



$$47 - 23$$

- First counting back in tens and ones.

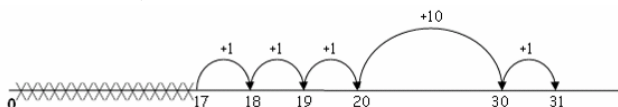


Counting on

$$31 - 17$$

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

- Count up from 17 to 20 (the next multiple of 10). Count up in tens to 30 (the multiple of ten before 31). Then count in ones to 31.



Building towards an efficient written method

Year 3 onwards

Using the empty number line

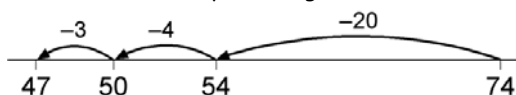
The empty number line

- The empty number line helps to record or explain the steps in mental calculation
- Either counting up or back may be used to record calculations
- The empty number line is a useful way of modelling processes such as bridging through a multiple of ten

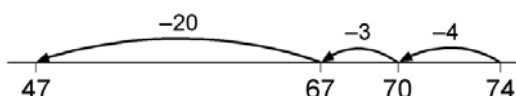
Counting-back method

- A calculation like $74 - 27$ can be recorded by counting back 27 from 74 to reach 47
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as $57 - 12$, $86 - 77$ or $43 - 28$

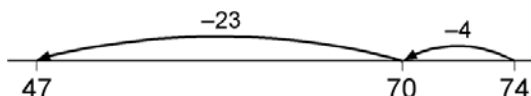
$74 - 27 = 47$ worked by counting back:



The steps may be recorded in a different order:



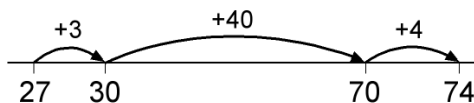
or combined:



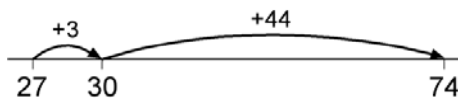
The counting-up method

- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically* in columns.
- The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as $30 + \square = 74$ mentally.

*Note: we would not advocate this method if moving onto compact decomposition



or:

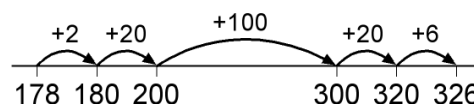


$$\begin{array}{r} 74 \\ - 27 \\ \hline 3 \rightarrow 30 \\ 40 \rightarrow 70 \\ \underline{4} \rightarrow 74 \\ 47 \end{array}$$

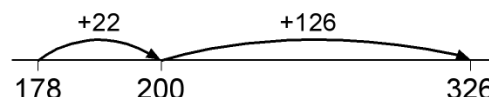


$$\begin{array}{r} 74 \\ - 27 \\ \hline 3 \rightarrow 30 \\ \underline{44} \rightarrow 74 \\ 47 \end{array}$$

- With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as $178 + \square = 200$ and $200 + \square = 326$ mentally.
- The most compact form of recording remains reasonably efficient.



or:

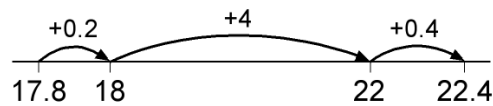


$$\begin{array}{r} 326 \\ - 178 \\ \hline 2 \rightarrow 180 \\ 20 \rightarrow 200 \\ 100 \rightarrow 300 \\ \underline{26} \rightarrow 326 \\ 148 \end{array}$$

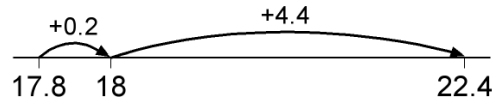


$$\begin{array}{r} 326 \\ - 178 \\ \hline 22 \rightarrow 200 \\ \underline{126} \rightarrow 326 \\ 148 \end{array}$$

- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.



or:



$$\begin{array}{r} 22.4 \\ -17.8 \\ \hline 0.2 \rightarrow 18 \\ 4.0 \rightarrow 22 \\ \hline 0.4 \rightarrow 22.4 \\ 4.6 \end{array}$$



$$\begin{array}{r} 22.4 \\ -17.8 \\ \hline 0.2 \rightarrow 18 \\ 4.4 \rightarrow 22.4 \\ 4.6 \end{array}$$

Partitioning

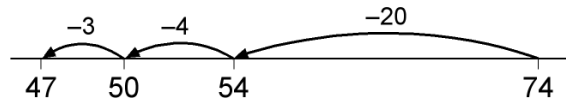
- Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For $74 - 27$ this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn. Some children may need to partition the 74 into $70 + 4$ or $60 + 14$ to help them carry out the subtraction.

Subtraction can be recorded using partitioning:

$$74 - 27 = 74 - 20 - 7 = 54 - 7 = 47$$

$$74 - 27 = 70 + 4 - 20 - 7 = 60 + 14 - 20 - 7 = 40 + 7$$

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.



Year 3 onwards

Expanded layout, leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Partitioned numbers are then written under one another:

Example: $74 - 27$

$$\begin{array}{r} 70 + 4 \\ - 20 + 7 \\ \hline 40 + 7 \end{array} \quad \begin{array}{r} 60 \quad 14 \\ \cancel{70} + 4 \\ - 20 + 7 \\ \hline 40 + 7 \end{array} \quad \begin{array}{r} 6 \quad 14 \\ \cancel{7} \quad 4 \\ - 2 \quad 7 \\ \hline 4 \quad 7 \end{array}$$

Example: $741 - 367$

$$\begin{array}{r} 700 + 40 + 1 \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array} \quad \begin{array}{r} 600 \quad 130 \quad 11 \\ \cancel{700} + 40 + 1 \\ - 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array} \quad \begin{array}{r} 6 \quad 13 \quad 11 \\ \cancel{7} \quad 4 \quad 1 \\ - 3 \quad 6 \quad 7 \\ \hline 3 \quad 7 \quad 4 \end{array}$$

Note: When partitioning numbers to subtract you may choose to use the symbol + or the word 'and'.

The expanded method for three-digit numbers - grading of difficulty

Example: $563 - 241$, no adjustment or decomposition needed

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 40 + 1 \\ \hline 300 + 20 + 2 \end{array}$$

$$\begin{array}{r} \text{leading to} \\ 563 \\ - 241 \\ \hline 322 \end{array}$$

Note: When partitioning numbers to subtract you may choose to use the symbol + or the word 'and'.

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

⇒ Example: $563 - 271$, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 70 + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 400 \quad 160 \\ \cancel{500} + \cancel{60} + 3 \\ - 200 + 70 + 1 \\ \hline 200 + 90 + 2 \end{array}$$

$$\begin{array}{r} 4 \quad 16 \\ \cancel{56}3 \\ - 271 \\ \hline 292 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how $500 + 60$ can be partitioned into $400 + 160$. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

⇒ Example: $563 - 278$, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500 + 60 + 3 \\ - 200 + 70 + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 400 \quad 150 \quad 13 \\ \cancel{500} + \cancel{60} + \cancel{3} \\ - 200 + 70 + 8 \\ \hline 200 + 80 + 5 \end{array}$$

$$\begin{array}{r} 4 \quad 15 \quad 1 \\ \cancel{56}3 \\ - 278 \\ \hline 285 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how $60 + 3$ is partitioned into $50 + 13$, and then how $500 + 50$ can be partitioned into $400 + 150$, and how this helps when subtracting.

⇒ Example: $503 - 278$, dealing with zeros when adjusting

$$\begin{array}{r} 500 + 0 + 3 \\ - 200 + 70 + 8 \\ \hline \end{array}$$

$$\begin{array}{r} 400 \quad 100 \quad 13 \\ \cancel{500} + \cancel{0} + \cancel{3} \\ - 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

$$\begin{array}{r} 4 \quad 9 \quad 1 \\ \cancel{50}3 \\ - 278 \\ \hline 225 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the $500 + 0$ is partitioned into $400 + 100$ and then the $100 + 3$ is partitioned into $90 + 13$.

Refining use of expanded layout, leading to column method, subtracting whole numbers and decimals with one or two places.

Example: $2362 - 548$

$$\begin{array}{r} 2300 + 60 + 2 \\ - 500 + 40 + 8 \\ \hline \end{array}$$

$$\begin{array}{r} \quad \quad \quad -50 \quad 12 \\ 2300 + \cancel{60} + \cancel{2} \\ - 500 + 40 + 8 \\ \hline 1800 + 10 + 4 \end{array}$$

$$\begin{array}{r} \quad \quad \quad 1 \quad 1 \quad 5 \quad 1 \\ \cancel{2}3\cancel{6}\cancel{2} \\ - 548 \\ \hline 1814 \end{array}$$

Example: $72.5 - 45.7$

$$\begin{array}{r} 11 \\ 60 + \cancel{1} + 15 \\ - 70 + \cancel{2} + 0.5 \\ - 40 + 5 + 0.7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ \cancel{72}.5 \\ - 45.7 \\ \hline 26.8 \end{array}$$